STAT 514 Midterm 1

Solution

1. Hypotheses: $H_0: \mu = \mu_0$ vs $H_1: \mu < \mu_0$

   Since population standard deviation $\sigma = 25$ is known, we should use Z-test with test statistic

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

   We reject H$_0$ if $Z < -Z_{0.05} = -1.645$, which is equivalent to

$$\overline{X} < \frac{-1.645 \times 25}{\sqrt{n}} + \mu_0$$

   So

$$\begin{aligned}
\beta &= \mathrm{P}(\text{fail to reject } H_0 \mid H_1) \\
&= \mathrm{P}\left(\overline{X} \geq \frac{-1.645 \times 25}{\sqrt{n}} + \mu_0 \mid H_1\right) \\
&= \mathrm{P}\left(\frac{\overline{X} - \mu_1}{25/\sqrt{n}} \geq \frac{\frac{-1.645 \times 25}{\sqrt{n}} + \mu_0 - \mu_1}{25/\sqrt{n}} \mid H_1\right) \qquad \text{under } H_1: \overline{X} \sim N(\mu_1, \frac{25}{\sqrt{n}})
\end{aligned}$$

   We need at least 80% power for 15 beats/minute reduction, so

$$\beta = \mathrm{P}\left(Z \geq \frac{\frac{-1.645 \times 25}{\sqrt{n}} + 15}{25/\sqrt{n}}\right) \leq 0.2$$

   Hence,

$$\begin{aligned}
&\frac{\frac{-1.645 \times 25}{\sqrt{n}} + 15}{25/\sqrt{n}} \geq 0.84 \\
&\frac{-1.645 \times 25}{\sqrt{n}} + 15 \geq \frac{0.84 \times 25}{\sqrt{n}} \\
&\frac{25}{\sqrt{n}}(1.645 + 0.84) \leq 15 \\
&n \geq \left[\frac{25}{15}(1.645 + 0.84)\right]^2 = 17.1534
\end{aligned}$$

   $\therefore$ At least 18 patients must be studied.

2. $\bar{y}_{1.} = 5.1, \quad \bar{y}_{2.} = 6.2, \quad \bar{y}_{3.} = 4.6, \quad n = 4, \quad a = 3, \quad SS_E = 2.3, \quad N = 12$

$$\therefore \bar{y}_{..} = \frac{(5.1+6.2+4.6)\times 4}{12} = 5.3$$

(a) ANOVA table:

Source	DF	Sum of Squares	Mean Square	F-value	Pr > F
Model	2	5.36	2.68	10.4851	< 0.01
Error	9	2.3	0.2556		
Total	11	7.66			

$$SS_T = \sum_{i=1}^{a} n_i(\bar{y}_{i.} - \bar{y}_{..})^2 = 4\left[(5.1-5.3)^2 + (6.2-5.3)^2 + (4.6-5.3)^2\right] = 5.36$$

(b) $H_0 : \mu_1 = \mu_2 = \mu_3$ (or $\tau_1 = \tau_2 = \tau_3 = 0$)

H_a : At least one is different.

Based on F-test, reject H_0 at $\alpha = 0.05$

(c) $R^2 = \frac{5.36}{7.66} = 0.6997$, which means 69.97% of the total variability can be explained by the model.

$$CV = \frac{\sqrt{MSE}}{\bar{y}_{..}} \times 100\% = \frac{\sqrt{.2556}}{5.3} \times 100\% = 9.539\%$$

(d) $\hat{\tau}_1 = \bar{y}_{1.} - \bar{y}_{..} = 5.1 - 5.3 = -0.2$

$\hat{\tau}_2 = \bar{y}_{2.} - \bar{y}_{..} = 6.2 - 5.3 = 0.9$

$\hat{\tau}_3 = \bar{y}_{3.} - \bar{y}_{..} = 4.6 - 5.3 = -0.7$

(e) e1) $\alpha' = \frac{\alpha}{2} = \frac{1-0.99}{2} = 0.005$

CI using Bonferroni: $\sum c_{ij}\bar{y}_{i.} \pm t_{\alpha/2m,(N-a)} \sqrt{MS_E \sum \frac{c_{ij}^2}{n_i}}$

CI for Γ_1: $(6.2-4.6) \pm t_{0.0025,9} \sqrt{0.2556(\frac{1}{4}+\frac{1}{4})}$

$= 1.6 \pm 3.69 \times 0.3575 = 1.6 \pm 1.3192$

$= (0.2802, 2.9192)$

CI for Γ_2: $(6.2+4.6-2\times 5.1) \pm t_{0.0025,9} \sqrt{0.2556(\frac{4}{4}+\frac{1}{4}+\frac{1}{4})}$

$$= 0.6 \pm 3.69 \times 0.6192 = 0.6 \pm 2.2848$$
$$= (-1.6848, 2.8848)$$

e2) CI using Scheffe: $\sum c_i \bar{y}_{i.} \pm \sqrt{(a-1)F_{\alpha,a-1,N-a}} \sqrt{MS_E \sum \frac{c_i^2}{n_i}}$

CI for Γ_1: $(6.2 - 4.6) \pm \sqrt{(3-1) \times F_{0.01,2,9}} \sqrt{0.2556(\frac{1}{4} + \frac{1}{4})}$

$$= 1.6 \pm \sqrt{2 \times 8.02} \times 0.3575 = 1.6 \pm 4.005 \times 0.3575 = 1.6 \pm 1.4318$$
$$= (0.1682, 3.0318)$$

CI for Γ_2: $(6.2 + 4.6 - 2 \times 5.1) \pm \sqrt{(3-1) \times F_{0.01,2,9}} \sqrt{0.2556(\frac{4}{4} + \frac{1}{4} + \frac{1}{4})}$

$$= 0.6 \pm 4.005 \times 0.6192 = 0.6 \pm 2.4799$$
$$= (-1.8799, 3.0799)$$

e3) Reject H_0 for Γ_1 but not Γ_2 using both methods.

Bonferroni is preferred since m=2, and Scheffe is too conservative when m is small.

3.

(a) ANOVA table:

Source	DF	Sum of Squares	Mean Square	F-value	Pr > F
Model	4	4939.3	1234.8	5.1244	< 0.01
Error	15	3614.5	240.9667		
Total	19	8553.8			

(b) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

H_a : At least one μ_i is different.

P-value < 0.01

Reject H_0.

(c) $\hat{\sigma}^2 = 240.9667$

(d) d1)

Contrast	df	SS	MS	F	Pr > F
C2	1	1817.160714	1817.160714	7.5411	0.01 < P < 0.025

d2) $\hat{C}2 = 2 \times 112.5 - 146.25 - 2 \times 149 - 158 + 2 \times 148.75 = -79.75$

$$\hat{\beta}_2 = \frac{\hat{C}2}{D_2} = \frac{-79.75}{2^2 + (-1)^2 + (-2)^2 + (-1)^2 + 2^2} = \frac{-79.75}{14} = -5.6964$$

Test $\beta_2 = 0$, $F_{20} = \frac{SS_{C2}}{MS_E} = \frac{1817.1607}{240.9667} = 7.5411$

P-value < 0.05

$\therefore$ The quadratic effect is significant at $\alpha = 0.05$.

d3) $SS_{C3} + SS_{C4} = SS_T - SS_{C1} - SS_{C2}$

$= 4939.3 - 2839.225 - 1817.1607$

$= 282.9143$

$$F_{30} \leq \frac{282.9143}{240.9667} = 1.1741$$

Since $F_{0.1,1,15} = 3.07$, neither can be significant.